Topology | Ph.D. Preliminary Exam | Aug. 22, 2001

Name:________________ ID No:____________ Section:______

Note : Do ONE and ONLY ONE of 6 and 7. Indicate clearly which one you chose.

1. Prove or disprove: The countable collection $\mathcal{B} = \{(a,b) | a < b, a \text{ and } b \text{ rational }\}$ is a basis that generates the standard topology on the real line R.

2. Let $\{A_\alpha | \alpha \in K\}$ be an arbitrary family of subsets of the topological space X. Prove or give a counterexample:
(a) $\overline{\cup_{\alpha \in K} A_\alpha} \subset \cup_{\alpha \in K} \overline{A_\alpha}$
(b) $\cup_{\alpha \in K} \overline{A_\alpha} \subset \overline{\cup_{\alpha \in K} A_\alpha}$

3. ~~Let $\{X_\alpha | \alpha \in K\}$ be an arbitrary family of topological spaces. If each X_α is Hausdorff,~~ then prove that the product space $\prod X_\alpha$ is also Hausdorff.

4. Consider the following subspace of the Euclidean plane:

$$S = \{(x, \sin 1/x) | 0 < x \leq 1\} \cup 0 \times [-1, 1]$$

. Prove or disprove the following statements:
(a) S is connected.
(b) S is path connected.

5. Prove that the Cartesian product of two compact space with the product topology is compact. Must show your work.

6. Define what it means for $p : X \longrightarrow Y$ to be a quotient map. State and prove two theorems about quotient maps. (The points awarded will depend on the difficulty and correctness of the proofs.)

7. A topological space is called "paracompact" if every open covering of X has locally finite refinement that covers X.
Prove that every paracompact space is normal.

Good Luck !!