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Algebra Preliminary Examination August 2002

1. Show that a group of order 150 has a proper normal subgroup.

2. (a) Let $n > 1$ be a positive integer. Show that every ideal in $Z/(n)$ is the product of maximal ideals.

(b) For which $n > 1$ is each ideal in $Z/(n)$ the product of distinct maximal ideals?

(c) For which $n > 1$ is $Z/(n)$ isomorphic to the product of fields?

3. (a) Find, up to isomorphism, all finite groups which are the union of exactly two conjugacy classes. (If G is a group and $x \in G$, then $\{gxg^{-1} \mid g \in G\}$ is the conjugacy class of x.)

(b) Find, up to isomorphism, all finite groups which are the union of exactly three conjugacy classes.

4. Let R be an integral domain and $0 \neq r \in R$.

(a) Define r is an irreducible in R.

(b) Define r is a prime in R.

(c) Prove or disprove: If $r \in R$ is irreducible, then it is prime.

(d) Prove or disprove: If $r \in R$ is prime, then it is an irreducible.