MATH 310 – Preliminary Examination

DEPARTMENT OF MATHEMATICS
University of Connecticut

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NAME: ______________________________

This exam has 5 pages including this cover.

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	40	
TOTAL	100	

1.) a) Prove that

$$||f||^2 = \int_{-1}^{1} x^2 (f^2 + 2(f')^2)\, dx$$

defines a norm in the function space $C^1[-1,1]$.

b) Prove that, for any continuously differential function f on $[-1,1]$,

$$\int_{-1}^{1} x^3 f(x) + 2x^2 f'(x) dx \leq \sqrt{\frac{26}{15}} \{ \int_{-1}^{1} (f^2 + 2(f')^2) dx \}^{1/2}.$$

2.) The *Volterra operator* V on $L^2(0,1)$ is the operator of indefinite integration:

$$(Vx)(t) = \int_0^t x(s)ds, \quad 0 < t < 1.$$

a) Prove that V is a bounded linear operator and that $||V|| \leq \frac{1}{\sqrt{2}}$.

b) Is this a compact operator? Give reasons.

3.) State and prove an existence and uniqueness theorem for the initial value problem

$$\begin{cases} y'' = xy^2 + e^y, & x \in (-\infty, \infty) \\ y(0) = y'(0) = 0. \end{cases} \tag{1}$$

4.) Consider the following Sturm-Liouville system

$$\begin{cases} (e^x f')' + \lambda e^x f = 0, & 0 < x < 1 \\ f(0) = f(1) = 0. \end{cases} \tag{2}$$

a) Find all eigenvalues and eigenfunctions;

b) Find the Green's function for the inhomogeneous problem

$$\begin{cases} f'' + f' = g(x), & 0 < x < 1 \\ f(0) = f(1) = 0. \end{cases} \tag{3}$$

c) Solve boundary value problem (3) for $g(x) = x$.

d) Solve the initial boundary value problem

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) + u_x(x,t), & 0 < x < 1 \\ u(0,t) = u(1,t) = 0, & t > 0 \\ u(x,0) = x, & 0 < x < 1. \end{cases} \tag{4}$$