Preliminary Exam – Risk Theory Section (Math 395)
August 26, 2002, 9AM

This is an open book examination. You may consult any printed source that you care to use. Recommended are Loss Models (Klugman, et al.) and Actuarial Mathematics, 2nd ed. (Bowers et al.). You may use any calculator you care to use. The questions are all from one extended problem, which may appear to be just a ruin theory problem. Solution, however, will require knowledge and techniques from across the range of topics covered in Math 395 Risk Theory.

An aggregate loss process $S(t) = X_1 + \dots + X_{N(t)}$ is a compound Poisson process with an average number of claims per year $E[N(1)] = 50$. The individual claim random variable X has the property that its associated per-loss excess loss variable $(X-d)_+$ has expected value $E[(X-d)_+] = 10{,}000\,[90{,}000/(d + 90{,}000)]^9$ for any d.

A premium process ct, when combined with this aggregate loss process, results in an adjustment coefficient R that satisfies the equation:

$$\int_0^{\infty} e^{Rx}\,(0.0001)[90{,}000/(x + 90{,}000)]^{10}\,dx = 3.75$$

The probability of ruin $\psi(u)$ with starting surplus u can be expressed in a formula that involves the cumulative probability distribution function $F_V(u)$ for a random variable $V = K_1 + \dots + K_M$ where M is a random counting variable and the K's are independent and identically distributed.

Questions:

1. What is the expected number of claims in 10 years?
2. What is the expected total claim amount in 20 years?
3. What is the distribution for the random variable M? Give both a formula (either for the probability function or for the cumulative probability distribution function) and a name for the distribution.
4. What is the distribution for the random variable K? Give both a formula (either for the probability density function or for the cumulative probability distribution function) and a name for the distribution.
5. If you approximate the distribution for K by a discrete distribution with a $10{,}000$ unit amount, and round all claim amounts to the nearest unit, what is the resulting approximate value for the probability of ruin $\psi(50{,}000)$ for an initial surplus of $50{,}000$?
6. Using the same approximation, what is the expected value for the largest deficit from a starting surplus of $50{,}000$ (i.e. largest negative value of $U(t) = ct + 50{,}000 - S(t)$) given that a deficit occurs?
7. What are the answers to 5 and 6 if the average number of claims per year is $E[N(1)] = 100$, with everything else staying the same?