307

Preliminary Exam — Topology — Aug 23. 04

Name:________________ ID No:____________ Section:______

1. Prove or disprove that every compact Hausdorff space is normal.

2. Let $[0,1] \times [0,1]$ be equipped with the dictionary order topology. Prove or disprove the following statements.

   a) $[0,1] \times [0,1]$ is connected.

   b) $[0,1] \times [0,1]$ is path connected.

3. Let X and Y be topological spaces and assume that $X \times Y$ has the product topology. Let $p : X \times Y \longrightarrow X$ be the projection. Prove or disprove each of the following statements:

   (a) p is open.

   (b) p is closed.

   (c) If X and Y are both connected, then $X \times Y$ is connected.

4. Let X be a **non-compact**, locally compact, Hausdorff topological space. Let $\{a, b\}$ be a two-point set such that $X \cap \{a, b\}$ is empty. Let $\mathcal{T}$ be a topology on $Y = X \cup \{a, b\}$ satisfying (1) Y is compact and connected with respect to $\mathcal{T}$ and (2) the subspace topology induced in X from $\mathcal{T}$ is the same as the original topology on X.

   a) Construct such a topology $\mathcal{T}$ on Y.

   b) Prove or disprove that $\mathcal{T}$ in (a) is Hausdorff.

   c) Prove or disprove that the intersection of any two compact subsets of Y is compact with respect to $\mathcal{T}$ in (a).

   d) Is such a $\mathcal{T}$ unique on Y up to a homeomorphism?

5. (a) Define what it means for a topological space to be compact (in terms of coverings by open sets).

   (b) Prove that X is compact if and only if every collection of closed sets in X with the finite intersection property has a nonvoid intersection.

thx. abe