Topology Preliminary Exam

Fall 2005
307

1. Prove that every infinite subset of a compact Hausdorff space has a limit point.

2. Prove that if X is a connected, countable, Hausdorff, normal space then X is a one-point space.

3. Let K and L be compact subsets of topological spaces X and Y, respectively. If W is an open set in $X \times Y$ with $K \times L \subset W$, show that there are open sets U in X and V in Y with $K \times L \subset U \times V \subset W$.

4. (a) Let A and B be subsets of a topological space X such that $A \cup B$ and $A \cap B$ are both connected. If A and B are both closed in X, prove that A and B are both connected.

   (b) Is the hypothesis that A and B be closed really needed? Prove or give a counterexample.

5. Given subsets A and B of connected spaces X and Y, respectively, with $A \neq X$ and $B \neq Y$, prove that $(X \times Y) - (A \times B)$ is connected.