# Topology

307- Prelim

August 2007

1. If $f : X \longrightarrow Y$ is a continuous map from a separable space $X$ onto a space $Y$, must $Y$ be separable? Prove or give a counter example.

2. Let $\{K_n | n \in N\}$ be a decreasing sequence of nonempty, compact subsets of a Hausdorff space $X$. If $U$ is an open set in $X$ with $\cap_{n=0}^{\infty} K_n \subset U$, prove that $K_n \subset U$ for some $n$. ($N$ denotes the set of positive integers.

3. Let $f : X \longrightarrow Y$ be a continuous map and let $G \doteq \{(x, y) \in X \times Y | \; y = f(x)\}$, where $G$ has the subspace topology inherited from $X \times Y$.

   (a) Prove that $X$ is homeomorphic to $G$.

   (b) If $Y$ is a Hausdorff space, then prove that $G$ is a closed subset of $X \times Y$.

4. Let $p : X \longrightarrow X/\sim$ be the quotient map induced by an equivalence relation $\sim$ on a space $X$. Suppose $\mathcal{T}$ is a topology on $X/\sim$ such that $p$ is continuous with respect to $\mathcal{T}$ and such that an arbitrary map $g : X/\sim \longrightarrow Y$ is continuous with respect to $\mathcal{T}$ precisely when its composite $g \circ p : X \longrightarrow Y$ is continuous. Must $\mathcal{T}$ be he quotient topology? Prove or disprove.

5. Let $A$ and $B$ be subsets of a topological space $X$ such that $A \cup B$ and $A \cap B$ are both connected.

   (a) If $A$ and $B$ are both closed subsets of $X$, prove that $A$ is connected.

   (b) Is the hypothesis that $A$ and $B$ be closed really needed to prove that $A$ is connected? Justify your answer.