5310 PRELIM
Introduction to Geometry and Topology
August 2012

You may use any result that has been proven in class, unless the question directly asks you to prove the result. Please, state the results you are using, and check that the assumptions are satisfied.

1. Show that $\mathbb{R}^2$ with the Euclidean metric topology coincides with the product topology of $\mathbb{R} \times \mathbb{R}$.

2. Suppose $f : X \to Y$ is a continuous bijection, X is compact, and Y is Hausdorff. Prove that f is a homeomorphism.

3. Let f be a continuous map between topological spaces X and Y. Prove or give a counterexample: if X is locally compact, that is, X is a Hausdorff space such that every point of X has an open neighbourhood whose closure is compact, then $f(X)$ is locally compact.

4. Determine the number of (not necessarily connected) $3:1$-coverings of $S^1 \times S^1$.

5. Let C be the union of two unlinked circles $\{(x, y, z) : (x-2)^2 + y^2 = 1, z = 0\}$ and $\{(x, y, z) : (x+2)^2 + y^2 = 1, z = 0\}$ in $\mathbb{R}^3$. Show that $\pi_1\left(\mathbb{R}^3 \backslash C\right)$ is a free group on two generators.

6. Suppose $p : Y \to X$ is a covering such that $p^{-1}(x)$ is finite and nonempty for all $x \in X$. Show that Y is compact Hausdorff if X is compact Hausdorff.