University of Connecticut
Department of Mathematics
Preliminary Exam - Risk Theory Section (Math 5637)
Friday August 22, 2014

There are 6 questions and they will be equally weighted. Show your derivations and calculations and state reasons that justify your steps. You may use any hand-held calculator. There are 3 hours scheduled for the exam and you may request an additional hour if you need it. Mark your ID number clearly on each blue book or page that you submit, but do not identify yourself by name.

1. The Hermite polynomials are defined to be the polynomials $H_n(x)$ that make the following expression true where $\phi(x) = e^{-\frac{1}{2}x^2}$ is the standard normal density:
$$\phi^{(n)}(x) = H_n(x)\,\phi(x)$$
Derive down the general formula, including the coefficients, for the polynomial $H_n(x)$.

2. Derive the Weibull random variable (density $\frac{1}{x}\tau\left(\frac{x}{\theta}\right)^{\tau} e^{-\left(\frac{x}{\theta}\right)^{\tau}}$) by using a maximum entropy principle followed by two transformations of the resulting random variable. Show the density and distributions functions that arise at each step in the derivation.

3. Use the surface interpretation for the first and second moments to derive recursive formulas for $\mathbb{E}\left[(X - k\Delta)\,|_{X>k\Delta}\right]$ and $\mathbb{E}\left[(X - k\Delta)^2\,|_{X>k\Delta}\right]$ for $k = 0, 1, 2, \ldots$ for a random variable X that is constant on each interval $[k\Delta, (k+1)\Delta)$, assuming you are given the probability values $\mathbb{P}\left[k\Delta \le X < (k+1)\Delta\right]$ for $k = 0, 1, 2, \ldots$ and the values $\mathbb{E}[X]$ and $\mathbb{E}\left[X^2\right]$.

4. For an $(a, b, 0)$ distribution:
   (a) what values of a and b are allowed, and why?
   (b) what does the the value of a tell us about the distribution?
   (c) derive the third moment of the distribution in terms of a and b.

5. Let N be a Poisson random variable with $\lambda = .8$, M a Binomial random variable with $q = .5$ and $m = 5$, and $L = M_1 + \ldots + M_N$ with M's all independent and independent of N. Calculate the probabilities that $L = 0, 1, 2, \ldots, 5$.

6. Derive an expression for the third moment of the contingent (i.e. "per payment") excess loss variable $(X - d)\,|_{X>d}$ in terms of moments and limited moments of X.