University of Connecticut
Department of Mathematics
Preliminary Exam - Risk Theory Section (Math 5637)
Friday, January 14, 2011

There are 5 questions. Show your calculations and state reasons that justify your steps, although you do not need to prove results formally except for question #3. A summary of key formulae for a variety of distributions is attached to this examination. You may use any hand-held calculator. There are 3 hours scheduled for the exam and you may request an additional hour if you need it. Mark your ID number clearly on each blue book or page that you submit, but do not identify yourself by name.

1. Write the 8-th central moment of the compound Poisson random variable $S = X_1 + ...X_N$ in terms of the variance of the Poisson random variable N and the raw moments of the i.i.d. random variables X_1, ... ,X_N.

2. Individual loss amounts ground up this year follow a distribution with mean 300 and standard deviation 300. This distribution of loss amounts is a power transformation of a shape-transformation of the distribution of a random variable X whose density function has maximum entropy on $[0, \infty)$ subject to the constraint that $\mathbb{E}[X] = 1$. Next year you expect loss amounts to inflate by 5% uniformly across all loss amounts. What is the coefficient of variation on a "per payment" basis next year for losses subject to a 100 deductible per loss and limited to $1,000$ per loss after the deductible is taken.

3. Assume that X has non-negative support and follows a continuous distribution for non-negative values. Prove that

$$TVaR_q(X) = \frac{\mu_X}{S_X(0)} + \int_{\pi_0}^{\pi_q} e_X(\pi_p) h_X(\pi_p) d\pi_p$$

where e_X is the mean excess loss function, h_X is the hazard rate function and π_p is the probability p quantile of X.

4. Consider a Poisson process $N(t)$ that produces losses at a rate of λ losses per year. Suppose individual losses are exponentially distributed with mean θ. Derive the cumulative distribution function $F_{M_N}(x)$ for the maximum loss M_N occuring in a period of k years.

5. Let $S(t) = X_1 + ...X_{N(t)}$ where $N(t)$ is Poisson with frequency $10t$ and the X's are indepedent and identically distributed with the property that the conditional distribution of $S(t)|N(t) = N^*$ is a gamma distribution with parameter $\alpha = N^*$ and mean $3N^*$ for any integer N^*. Let $L = Max_{t \geq 0} \{(S(t) - 33t)_+\}$ be the maximum agregate loss random variable with premium rate $c = 33$. Express L as $L = K_1 + ... + K_M$ where M is

a random counting variable and the K's are independent and identically distributed. Approximate K (by rounding) using a discrete distribution with whole integer units. Calculate the resulting approximate values for (a) the probability $\psi(2)$ of ruin from a starting surplus of 2 and (b) the expected value $\mathbb{E}\left[(L-2)\,|L>2\right]$ of the largest excess of accumulated claims over accumulated premiums plus starting surplus of 2, contingent upon ruin occurring.