University of Connecticut
Department of Mathematics
Preliminary Exam - Risk Theory Section (Math 5637)
Monday, January 21, 2013

There are 6 questions. Show your derivations and calculations and state reasons that justify your steps. You may use any hand-held calculator. There are 3 hours scheduled for the exam and you may request an additional hour if you need it. Mark your ID number clearly on each blue book or page that you submit, but do not identify yourself by name.

1. Derive an expression for the 6th central moment of a random variable in terms of its cumulant moments κ_j for $1 \le j \le 6$.

2. Derive an expression for the third moment of the contingent (i.e. "per payment") excess loss variable $(X-d)\,|_{X>d}$ in terms of moments and limited moments of X.

3. For a random variable X with continuous support derive expressions for the derivatives of VaR_q and CTE_q with respect to q, the probability level of the risk. What conclusion can you draw about VaR compared to CTE as a measure of risk?

4. Derive the two-parameter Pareto distribution $F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha}$ from a maximum entropy principle followed by a series of transformations.

5. Let $S = M_1 + ... + M_N$ where N is Poisson $\lambda = .75$ and $\{M_j\}$ are i.i.d. Binomial $m = 4$, $q = .25$ and independent of N. Calculate the probability that $S > 4$. Be accurate to at least 3 significant digits.

6. Let K be Binomial (m, q), L be Poisson λ, and M be Negative Binomial (r, β) representing independent frequency of event variables. Suppose that in each case 25% of the events that actually occur fail to be recorded. Derive and identify (i.e. give their names and parameters) the corresponding frequency of recorded events variables, calling them K^*, L^*, and M^*.