# Risk Theory Preliminary Exam

January 17, 2014

There are six questions that will be equally weighted in the grading. You can use any hand-held calculator. Put your ID# on all blue books and on any scratch paper used. You have 3 hours to complete the exam.

1. For a Poisson random variable with frequency $\lambda$ what is the smallest integer $n$ for which the $n$-th central moment $\mu_n$ contains a term involving $\lambda^4$? Demonstrate why this is the smallest such $n$. What is the coefficient of $\lambda^4$ in the expression for $\mu_n$?

2. An event frequency variable is known to follow a Poisson distribution for each risk exposed to the event. The Poisson freqency $\lambda$ varies, however, among the $600,000$ risks exposed to the event. In fact, $\lambda$ is distributed across the entire population of risks as a sum of twenty-five independent *gamma* random variables, all having the same $\theta$ (or $\beta$) parameters. The first one has an $\alpha$ parameter of 0.2, the second 0.4, and so one with the twenty-fifth having an $\alpha$ parameter of 5.0. Next, suppose that the population of risks grows from $600,000$ to $1,000,000$ but does so in such a way that the aggregate event probability distribution remains relatively unchanged. Finally, suppose that 20% of all events are too small to be observed. What are the mean, variance, third and fourth central moments of the observed event frequency for the new population of $1,000,000$?

3. How is the two-parameter Pareto distribution analogous to the scaled Exponential distribution? Similarly, how is the Generalized Pareto distribution analogous to the scaled Gamma distribution? Give details.

4. If $N$ is a compound Poisson-Poisson random variable with parameters $\lambda = .25$ for the primary variable and $\lambda = 1$ for the secondary variable, and if $X$ can be approximated by a negative binomial random variable with $\beta = 4$ and $r = .25$ then calculate numerical values for the first 5 probabilities $\mathbb{P}[S = 0]$, $\mathbb{P}[S = 1]$, ..., $\mathbb{P}[S = 4]$ for the random variable $S = X_1 + ... + X_N$ with the $X_j$ being i.i.d. copies of the random variable $X$.

5. Calculate the logarithmic derivatives $\frac{d}{dq} \ln VaR(q)$ and $\frac{d}{dq} \ln CTE(q)$ of the Value at Risk and the Conditional Tail Expectation with respect to the probability level $q$. Does this suggest any conclusion about the relative merits of Value at Risk and Conditional Tail Expectation as measures of risk?

6. Consider a Poisson process $N(t)$ that produces losses at a rate of $\lambda$ losses per year. Suppose the individual losses are are exponentially distributed with mean $\theta$. Derive the cumulative distribution function $F_{M_N}(x)$ for the maximum individual loss $M_N$ occurring over a period of $k$ years.