STUDY GUIDE FOR PH.D EXAM IN REAL ANALYSIS

Basics of abstract analysis.

- Sequences and series of functions on compact metric spaces, uniform convergence, equicontinuous families on $C(K)$ and Arzelà-Ascoli theorem. Algebras of functions that separate points and Stone-Weierstrass theorem in $C(K)$.

Abstract integration (Lebesgue integration theory).

- Measure spaces and measurable functions.
- Outer measures and Carathéodory's theorem.
- Convergence theorems: Fatou's Lemma, Monotone and Dominated Convergence Theorems.
- Product measures. Fubini's and Tonelli's theorems.
- Signed measures and complex measures. Hahn-Jordan decomposition.
- Lebesgue decomposition. Radon-Nikodym Theorem.
- Modes of convergence and how they are related: uniform, pointwise, almost everywhere, in measure, in L^p-norm.
- Duality: Riesz Representation Theorem for bounded linear functionals on $C(K)$.

Integration on $\mathbb{R}^n$.

- Lebesgue measure on $\mathbb{R}^n$. Borel and Lebesgue σ-algebras. Non-measurable sets.
- Borel measures on $\mathbb{R}$ and their completion (Lebesgue-Stieltjes measures).
- Functions of bounded variation on $\mathbb{R}$ and absolutely continuous functions on $\mathbb{R}$. Riemann-Stieltjes integral.
- Characterization of Riemann integrability and Riemann integration through Lebesgue theory. Lebesgue Differentiation Theorem. Fundamental Theorem of Calculus for Lebesgue integrals.

L^p-spaces.

- Basic convexity inequalities: Hölder (including Cauchy-Schwarz), Minkowski, Jensen.
- Completeness. Separability.
- Duality: Riesz Representation Theorem for bounded linear functionals on L^p.

References.

Real Analysis and Probability, R.M. Dudley.
Real Analysis: Modern Techniques and Their Applications, G. Folland.
Real and Complex Analysis, W. Rudin.
Real Analysis, H.M. Royden.
Measure and Integral, R.L. Wheeden and A. Zygmund.

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