Notation and conventions:

- $\mathbb{D} = \{ z \in \mathbb{C} : |z| < 1 \}$ is the open unit disk.
- The terminology analytic function and holomorphic function may be used interchangeably.

**Problem 1.** How many solutions (counted with multiplicity) does the equation $z^6 + 5z^3 + 1 = 0$ have in the unit disk $\mathbb{D}$?

**Problem 2.** Let $f$ be a holomorphic map of the unit disk $\mathbb{D}$ into itself. Suppose $f$ is not the identity map. Can $f$ have two or more fixed points? Prove your assertion. (Recall $w \in \mathbb{D}$ is a fixed point of $f$ if $f(w) = w$.)

**Problem 3.** Prove or disprove that there exists a holomorphic function $f(z)$ defined on the punctured disk $\mathbb{D} \setminus \{0\}$ such that
\[ \lim_{z \to 0} zf(z) = 0 \quad \text{and} \quad \lim_{z \to 0} |f(z)| = \infty. \]

**Problem 4.** Find a one-to-one conformal map from $U = \{ z \in \mathbb{C} : |z| > 1 \text{ and } \Im(z) > 0 \}$ onto the unit disk $\mathbb{D}$.

**Problem 5.** Suppose $f$ is a non-constant holomorphic function on $\mathbb{D}$. Suppose $|f|$ is constant on the circle $|z| = \frac{1}{2}$. Show that $f$ has at least one zero in $\Omega = \{ z \in \mathbb{C} : |z| < \frac{1}{2} \}$.

**Problem 6.** Let $a$ be a positive real number. Compute
\[ \int_{-\infty}^{\infty} \frac{\cos(ax)}{(1 + x^2)^2} \, dx. \]

**Problem 7.** Is there a one-to-one conformal map from the punctured disk $\mathbb{D} \setminus \{0\}$ onto the annulus $A = \{ z \in \mathbb{C} : 1 < |z| < 2 \}$? Prove your assertion.