**Topology Prelim, August 2019**

1. Let $X$ be a topological space and $A, B$ be subsets of $X$. Prove or disprove the following set equalities.
(a) $\overline{X \setminus (A \cup B)} = X \setminus (IntA \cup IntB)$.
(b) $Int(X \setminus (A \cup B)) = X \setminus (\overline{A} \cup \overline{B})$.

2. Define the equivalence relation on $\mathbb{R}$ such that $x \sim y$ if $x - y$ is rational. Let $\mathbb{R}/\sim$ be the quotient space with the quotient topology. Show that $\mathbb{R}/\sim$ is not Hausdorff.

3. Let $A$ be an open subset in $\mathbb{R}^n, n \geq 2$, whose boundary $\partial A$ is connected. Is $\partial A$ necessarily path-connected? Prove your assertion.

4. Show that if a path-connected, locally path-connected space $X$ has $\pi_1(X)$ finite, then every map $X \to S^1$ is nullhomotopic, that is, it is homotopic to a constant map.

5. Compute the fundamental groups of the following spaces:
(a) $X \subset \mathbb{R}^3$ is the complement of the union of $n$ lines through the origin.
(b) $\mathbb{T}^2 \setminus \{p\}$, where $\mathbb{T}^2$ is the torus.

6. Prove that every continuous map $h : D \to D$ has a fixed point, that is, a point $x$ with $h(x) = x$. Here $D$ is the closed unit disk in $\mathbb{R}^2$.