University of Connecticut
Department of Mathematics
Preliminary Exam - Risk Theory Section (Math 5637)
Wednesday, August 25, 2010

There are 5 questions. Show your calculations and state reasons that justify your steps, although you do not need to prove results formally. A summary of key formulae for a variety of distributions is attached to this examination. You may use any hand-held calculator. There are 3 hours scheduled for the exam and you may request an additional hour if you need it. Mark your candidate number clearly on each blue book or page that you submit, but do not identify yourself in any other way.

1. What are the mean and second, third and fourth central moments of the Inverse Gaussian distribution with parameters μ and θ? Faá's formula might simplify the calculation for you.

2. Let X be the random variable whose density function has maximum entropy on $[0, \infty]$ subject to the constraint that $\mathbb{E}[X] = \mu$. Let Y be the random variable resulting from a shaping transformation of X with shaping parameter α. What are the density function, distribution function, name and parameters of the distribution for Y?

3. Individual loss amounts (ground up) this year follow a two-parameter Pareto distribution with mean 2 and standard deviation $\sqrt{12}$. Next year you confidently expect loss amounts to inflate by 10% uniformly across the board. What will be the standard deviation next year for loss amounts that are subject to 0.2 deductible per loss, with losses after deductible limited to 4 per loss? Please answer for the "per loss" variable, not the "per payment" variable.

4. A surplus process is defined by $u(t) = u + ct - S(t)$ where $S(t)$ is a compound Poisson process with $\lambda = 600$ and individual claim distribution $p(x) = 3e^{-3x}$. The premium accumulation rate is $c = 200$. For what value of intitial surplus u is the value of $\frac{d\psi(u)}{du}$ a maximum?

5. Let $S(t) = X_1 + ... + X_{N(t)}$ where $N(t)$ is Poisson with frequency $10t$ and the X's are independent and identically distributed with the property that the conditional distribution of $S(t)|(N(t) = N^*)$ is a Gamma distribution with parameter $\alpha = N^*$ and mean $2N^*$ for any integer N^*. Let $L = \max_{t \geq 0} \left\{ (S(t) - 22t)_+ \right\}$ be the maximum aggregate loss random variable with premium rate $c = 22$. Express L as $L = K_1 + ... + K_M$ where M is a random counting variable and the K's are independent and identically distributed. Approximate K (rounding) using a discrete distribution with whole integer units. Calculate the resulting approximate values for (a) the probability $\psi(2)$ of ruin from a starting surplus of 2 and (b) the conditional tail expectation $\mathbb{E}[(L - 2)_+ | L > 2]$.