Study Guide for Risk Theory Prelim (MATH5637)

1. Modeling with random variables
    a. pf, pdf, cdf, ddf, hazard rate, moments (and related measures), quantiles
    b. generating functions and transforms: moment-, probability-, cumulant-; Fourier (characteristic), Laplace; relations among them; uses; Faá formula and applications
    c. censored, truncated and/or shifted variables: contingent (per payment) variables, excess loss variables, non-contingent (per loss) variables, limited loss variables
    d. relationships among items in c. and their moments; surface interpretation: $E[X^k] = (-1)^k k \int_{-\infty}^{0} |x|^{k-1} F_X(x)dx + k \int_0^{\infty} x^{k-1} S_X(x)\, dx$; manipulation of weighted areas
2. Risk Assessment
    a. tail ranking by moments, limiting behavior, hazard rate and mean excess loss functions
    b. equilibrium distribution
    c. risk measures and coherence, VaR, TVaR (CTE)
3. Severity Models (continuous)
    a. maximum entropy principle; transformations: power, exponentiation, shaping, conditional tail moment, mixing, splicing, inversion; frailty
    b. families: transformed gamma, transformed beta, Gaussian, linear exponential, elliptical
    c. extreme value distributions; distribution & stability of the maximum; Fisher-Tippett; stability & distribution of excesses; Balkema- de Haan-Pickands
    d. multivariate models; Sklar theorem and copulas, dependency (Spearman, Kendall) and tail dependence; archimedean, elliptical, and extreme value copulas
4. Frequency Models and Processes (discrete)
    a. (a,b,0) distributions; truncation and modification, (a,b,1) distributions
    b. compound models, convolution, Panjer recursion, compound Poisson, mixed models, mixed Poisson, discrete linear exponential, TVar(CTE)
    c. exposure and infinite divisibility
    d. counting processes, Poisson processes, contagion
5. Effects of coverage modifications on Severity and Frequency Models
6. Collective Risk Models
    a. basic choices; compound risk models; analytic results
    b. calculation methods: approximation, convolution, Panjer recursion, Fast Fourier inversion, numerical inversion, TVar(CTE) methods
7. Individual Risk Models; parametric approximation; compound Poisson approximation
8. Ruin Models
    a. continuous-time Poisson process, adjustment coefficient, ruin probability: exact expression and Lundberg's inequality
    b. joint distribution of initial-reserve and deficit-at-ruin & its integro-differential equation
    c. survival probability, renewal equation, equilibrium distribution, maximum aggregate loss, compound geometric solution, Erlang mixture, Cramér's asymptotic approximation

References:

Primary: Loss Models (3rd ed.), Klugman et al. ch.1-9,11;

Math 5637 classroom notes and notes/examples at http://www.math.uconn.edu/~bridgeman/math5637/;

In addition, see: Actuarial Mathematics (2nd ed.), Bowers et al. ch.13, including appendix;

H. Gerber & E. Shiu, On the Time Value of Ruin, *North American Actuarial Journal* v. 2 #1 (Jan. 1998) pp. 48-78

K. Conrad, http://www.math.uconn.edu/~kconrad/blurbs/analysis/entropypost.pdf;

H. Flanders, From Ford to Faá, *American Mathematical Monthly* v.108 #6 (June-July 2001) pp. 559-61