Loss Models Prelims for Actuarial Students
August 23, 2021 MONT 313, 9:00 am - 1:00 pm

Instructions:

1. There are five (5) questions here and you are to answer all five. Each question is worth 20 points.
2. Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
3. Please write legibly. Points will be deducted for incoherent, incorrect, and/or irrelevant statements.

Question No. 1:

Let $\{X_i\,;\, i = 1, 2, \cdots, n\}$ be a series of independent and identically distributed (i.i.d.) random variables, where X_i has the same distribution as a **zero-truncated** binomial distribution $\mathcal{BN}(4, 0.25)$. Define S by the following compound Poisson model

$$S := \sum_{i=1}^{N} X_i = X_1 + X_2 + \cdots + X_N$$

with the usual convention that $S = 0$ if $N = 0$, where N follows a Poisson $\mathcal{PN}(2)$ distribution. Assume N and all X_i's are independent.

(a) Find the probability function of X_i.

(b) Calculate the mean and variance of S.

(c) Calculate the probability that S is no greater than 1, that is, $\Pr(S \leq 1)$.

Note: Please round the final answers to 6 decimal places.

Question No. 2:

Suppose the claim severity X is modeled with its pdf given by

$$f_X(x) = \frac{c}{x^3}, \quad \text{for } 1 \leq x \leq 4,$$

and 0 otherwise.

(a) Calculate the 90% quantile of X ($x_{90\%}$).
Hint: $x_{90\%}$ is defined as $\Pr(X \leq x_{90\%}) = 90\%$.

(b) Assume there is a deductible of 2 applied to each claim, and denote the after-deductible claim size by Y (i.e. $Y := (X - 2)_+$). Find the probability density function (pdf) of Y.

Question No. 3:

Suppose you are an actuary interested in recent occurrence of cyber attacks and you have data from companies with insurance coverage for cyber attacks. The observations are $X_1, X_2, \ldots, X_n$ drawn from a random sample with a Bernoulli probability function

$$\Pr(X = k) = p^k (1-p)^{1-k}, \quad \text{for } k = 0, 1$$

where p represents the probability that a cyber attack occurs and $0 \le p \le \frac{1}{2}$.

(a) Derive the method of moment estimator of p, and denote it as $\tilde{p}$. Comment on the unbiasedness of this estimator.

(b) Simplify an expression for the mean squared error of $\tilde{p}$.

(c) Derive the maximum likelihood estimator of p, and denote it as $\widehat{p}$. Comment on the unbiasedness of this estimator. **Hint**: $\tilde{p}$ is not the same as $\widehat{p}$.

(d) Find the mean squared error of $\widehat{p}$. Simplify this expression as much as possible.

(e) Compare the two estimators and state which estimator is preferred. Justify your choice.

Question No. 4:

Insurance policies have deductibles d, maximum covered losses of 15, and ground-up losses x. Twenty observed losses, $i = 1, 2, \ldots, 20$, are recorded below:

i	d_i	x_i	i	d_i	x_i	i	d_i	x_i	i	d_i	x_i
1	0	12	6	2	14	11	4	13	16	5	10
2	0	16	7	2	13	12	4	18	17	5	9
3	0	4	8	2	17	13	4	7	18	5	8
4	0	12	9	2	9	14	4	12	19	5	16
5	0	15	10	2	8	15	4	18	20	5	13

(a) Compute the risk set of each observed ground-up loss amount and estimate the probability of loss exceeding 10 using the Kaplan-Meier method.

(b) Compute the 95% linear confidence interval of the probability of loss in (a). Explain a disadvantage of this approach and what has been suggested to overcome this drawback?

(c) If the loss observations are grouped into the intervals $(0, 5]$, $(5, 10]$, and $(10, 15]$, determine the risk set in each interval. Estimate the probability of loss exceeding 10 based on this grouped loss data.

(d) Explain disadvantage(s) of using grouped data for estimation.

Question No. 5:

Suppose you are evaluating linear models to a dataset with $n = 500$ observations. The table below shows you six different fitted models, with $\log L(\widehat{\theta}; \mathbf{x})$ as the log-likelihood and p as the corresponding number of model parameters.

Model	p	$\log L(\widehat{\theta}; \mathbf{x})$
1	3	-879.43
2	3	-1404.82
3	3	-1407.41
4	4	-876.26
5	5	-866.45
6	9	-865.49

(a) Define AIC and compute the AIC for each of the fitted models above.

(b) Define BIC and compute the BIC for each of the fitted models above.

(c) Discuss the purposes of using information criteria for model selection.

(d) Compare the two performance measures, AIC and BIC, for model selection.

(e) Choose the <u>best</u> model among the six fitted models above. Justify your choice.

—— end of exam ——

APPENDIX

A random variable X is said to have a Gamma distribution with scale parameter $a > 0$ if its density has the form

$$f(x) = \frac{a^b x^{b-1} e^{-ax}}{\Gamma(b)}, \quad \text{for } x > 0.$$

A random variable X is said to be a two-parameter Pareto(α, θ) if its cumulative distribution function has the form

$$F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha}, \quad \text{for } x > 0.$$

Its mean and variance are, respectively,

$$\mathrm{E}(x) = \frac{\theta}{\alpha - 1} \quad \text{and} \quad \mathrm{Var}(X) = \frac{\alpha\theta^2}{(\alpha-1)(\alpha-2)},$$

provided they exist.