1. Let $X$ be a Hausdorff space and $A \subset X$ a closed subset.
   (a) Prove that the quotient map $\pi : X \to X/A$ is a closed map.
   (b) Prove that if $X$ is normal, then the quotient space $X/A$ is normal.

   Recall that $Z$ is called normal if for any disjoint closed sets $C, D \subset Z$, there exist disjoint open sets $U, V \subset Z$ with $C \subset U$ and $D \subset V$.

2. Let $X$ and $Y$ be topological spaces and consider the product space $X \times Y$. Let $A$ be a compact subset of $X$ and $B$ be a compact subset of $Y$. Prove that if $W$ is an open set containing $A \times B$, then there exists $U$ open in $X$ and $V$ open in $Y$ such that $A \times B \subset U \times V \subset W$.

3. Let $X$ be a connected space and $f, g : X \to [0, 1]$ continuous functions with $f$ surjective. Prove there exists $x \in X$ such that $f(x) = g(x)$.

4. Let $X$ be a topological space and $A \subset X$ a deformation retract of $X$. Prove that if $A$ is path connected, then $X$ is path connected as well.

5. Compute the fundamental group of $X$, where:
   (a) $X$ is the complement of $n$ distinct points on the cylinder $\mathbb{R} \times S^1$.
   (b) $X$ is the complement in $\mathbb{R}^3$ of $n$ distinct lines through the origin.

6. Let $A$ be a path connected subset of a path connected space $X$ and let $i : A \to X$ be the inclusion map of $A$ into $X$. Assume $X$ admits universal cover and let $p : \tilde{X} \to X$ be the universal covering map.
   Prove that $p^{-1}(A)$ is path connected if and only if $i_* : \pi_1(A, a) \to \pi_1(X, a)$ is surjective, for any $a \in A$. 

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