Loss Models Prelims for Actuarial Students
August 2025

Instructions:

1. There are five (5) equally-weighted questions and you are to answer all five.
2. Hand-held calculators are permitted.
3. Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
4. Please write legibly. Points will be deducted for incoherent, incorrect, and/or irrelevant statements.

Question No. 1:

Consider a collective risk model

$$S = \sum_{i=1}^{N} X_i,$$

in which N follows a Poisson distribution with mean 4 (i.e., $N \sim \mathcal{PN}(4)$), and the i.i.d. losses X follow a binomial distribution, $X \sim \mathcal{BN}(1, 0.2)$.

Use **two different methods** to calculate the probabilities that S equals 0 and 1, $\mathbb{P}(S = 0)$ and $\mathbb{P}(S = 1)$.

Question No. 2:

Suppose the pf-pdf of a non-negative random variable X is given by

$$f_X(x) = \begin{cases} 0.45, & \text{if } x = 0 \\ \dfrac{1}{18} x^2, & \text{if } x \in (0, 3) \\ p, & \text{if } x = 3 \\ 0, & \text{otherwise} \end{cases},$$

in which p is a positive constant yet to be determined.

(a) Calculate $\mathrm{VaR}_{90\%}(X)$.

(b) Calculate $\mathrm{CVaR}_{90\%}(X)$.

Hint: Denoting F_X the cdf of a given random variable X, we define $\mathrm{VaR}_\delta(X) := \inf\{x \in \mathbb{R} : F_X(x) \geq \delta\}$ and $\mathrm{CVaR}_\delta(X) := \frac{1}{1-\delta} \int_\delta^1 \mathrm{VaR}_\xi \, \mathrm{d}\xi$, for all $\delta \in (0, 1)$.

Question No. 3:

Suppose that the aggregate loss S is given by the following collective risk model:

- the claim frequency N follows a Poisson $\mathcal{PN}(100)$ distribution;

- the (i.i.d.) ground-up losses X_is follow an exponential distribution with mean 200.

Now all the policies are modified with a deductible of $d = 50$ and a maximum **covered loss** of $u = 350$. Denote the aggregate loss after modifications, $\tilde{S}$, by

$$\tilde{S} = \sum_{i=1}^{\tilde{N}} \tilde{X}_i,$$

in which $\tilde{N}$ and $\tilde{X}_i$s are modified versions of N and X_is. That is, $\tilde{X}_i > 0$ is the payment size in the i-th **payment event**, and $\tilde{N}$ counts all the payment events.

(a) Calculate the mean and second moment of $\tilde{X}$: $\mathbb{E}[\tilde{X}]$ and $\mathbb{E}[\tilde{X}^2]$.

(b) Calculate the mean and variance of $\tilde{S}$: $\mathbb{E}[\tilde{S}]$ and $\mathbb{V}[\tilde{S}]$.

Question No. 4:

The Zero-Inflated Poisson (ZIP) model is becoming widely used to model claim frequency N to account for data with excess zeros relative to the standard Poisson distribution. It assumes that zeros arise from two sources:

(i) a degenerate process that always yields zero values; and

(ii) a standard Poisson process.

Let $Y \sim \text{ZIP}(\pi, \lambda)$ where

- $\pi \in [0, 1]$ is the probability of an excess zero;
- $\lambda > 0$ is the standard Poisson mean parameter; and
- hence, the probability mass function is expressed as

$$p_n = \mathbb{P}(N = n) = \begin{cases} \pi + (1-\pi)e^{-\lambda}, & \text{if } n = 0, \\ (1-\pi)\dfrac{\lambda^n e^{-\lambda}}{n!}, & \text{if } n > 0. \end{cases}$$

(a) Derive the mean and variance of the ZIP distribution in terms of the parameters π and λ.

(b) Show that the variance of the ZIP distribution exceeds the mean unless $\pi = 0$. Interpret this in the context of overdispersion.

(c) Now, suppose you observe an i.i.d. sample of m observations $n_1, n_2, \ldots, n_m$ from a ZIP(π, λ).

  (i) Write the log-likelihood function $\ell(\pi, \lambda)$.

  (ii) Derive the score equations for the maximum likelihood estimation (MLE). Simplify your equations using the following symbols:

  N_0 = total number of zeros observed; and

  $N_+ = N - N_0$ = total number of positive counts observed.

  Do not solve.

(iii) It will be clear there is no explicit solutions for the MLE. Suggest two possible ways to approximate the MLE in this case.

Question No. 5:

Suppose the pdf of the loss random variable X is given by

$$f(x) = \frac{8\theta^2}{x^3}, \qquad x \geq 2\theta,$$

in which $\theta > 0$ is an unknown parameter. A random sample of 5 observations on X is obtained:

$$x_1 = 8,\ x_2 = 10,\ x_3 = 12,\ x_4 = 14,\ x_5 = 16.$$

(a) Use the quantile matching method (matching the median) to estimate θ.

(b) Use the moments matching method (matching the mean) to estimate θ.

— end —